

A sampler of problems on Proportional and Linear Relationships for Math 5035/7035, Algebra for Middle School Teachers.

1. Ed has just arrive at college, and wants to open up a checking account. He asks his older sister Sally how much, if anything, her bank charges as a monthly maintenance fee, and how much, if any, it charges for each check written. Sally doesn't remember, but gets out her last two statements. She sees that in June she wrote 13 checks, and had a total monthly service fee of \$3.26, while in July she wrote 21 checks and had a total monthly service fee of \$3.82. Does Sally have enough information to answer Ed's questions?
2. Come to class tomorrow with a list of at least six variable quantities, and two constant quantities, that you have encountered during the day.
3. Carla has been keeping track of her monthly telephone long-distance charges, and has made the following table:

<u>Month</u>	<u>Minutes</u>	<u>Charge</u>
Jan	123	\$9.84
Feb	168	\$13.44
Mar	98	\$7.84
Apr	115	\$9.20

Explain her long-distance telephone company's fee structure.

4. Prepare a table similar to Carla's, for any four consecutive months of your own long-distance telephone charges. Can you explain your long-distance telephone company's fee structure?
5. Find two pairs of geometric quantities that are related to each other proportionally, and two other pairs of geometric quantities that are related to each other, but not proportionally.
6. How would you explain to a middle school student what division means? Does your explanation encompass "1 divided by 3", that is, $1/3$? Does your explanation explain why $1/(1/3)=3$? Does your explanation explain why $a/(b+c) = a/b + a/c$?
7. Simon and Garfunkel High School has noticed over the years that when they announce soccer tryouts, 45% of the students show up, and of these, $2/3$ are boys.
 - (a) One year, 324 students showed up for tryouts. How many students were in the school that year?
 - (b) Draw a graph whose horizontal axis represents the number of boys who show up for practice, and whose vertical axis represents the number of students in the school. What is the slope of this graph?

8. Designers seem to believe that the most pleasing rectangles to look at have lengths that are about 1.62 times their heights. Find formulas for:
 - (a) The area of such a rectangle in terms of its height.
 - (b) The perimeter of such a rectangle in terms of its length.
9. If quantities A and B are proportional, and quantities B and C are proportional, are quantities A and C necessarily proportional? Explain.
10. Carla's twin sister Carly received a mail solicitation from a long-distance telephone company. She didn't read the fine print explaining the fee structure, but she did notice the following simple chart:

<u>Minutes used</u>	<u>monthly charge</u>
15	\$5.20
30	\$7.15
45	\$9.10
60	\$11.05

Based on the above fee structure, what would Carly's monthly charge be if she talked for 75 minutes?

11. Who has a better long-distance plan, Carla or Carly? Explain.
12. John can walk at a speed of 8 mph, while his elderly father can only walk at a speed of 2 mph.
 - (a) In the time it takes his father to walk 3 miles, how far can John walk?
 - (b) After they have walked for 2 hours, how far ahead will John be?
13. Come to class tomorrow with a list of five variable quantities, two of which have increased in value between this morning and this evening, two of which have decreased in value between this morning and this evening, and one of which has stayed constant between this morning and this evening.
14. How would you explain to a middle school student what putting a minus sign in front of a number means? Does your explanation explain why $-(-4)=4$?
15. A pizza parlor has fixed monthly costs (such as rent) and variable monthly costs (such as the purchase of ingredients, which depends upon the number of pizzas sold). Draw a graph whose horizontal axis represents the number of pizzas sold in a month, and whose vertical axis represents the total expenses of the pizza parlor. Suppose that the fixed costs increased, but the variable costs decreased. What would the new graph look like?
16. Let (3,7) and (5,11) be two points on a line. Find the formula for the line by first computing the slope, and then using
 - (a) the slope and the point (3,7)
 - (b) the slope and the point (5,11).
 Do you get the same equation? Explain.

17. Think of a proportional rule, two different linear (but not proportional) rules, and one non-linear rule, that each take the number 1 and assign it a value of 2.
18. Carl was offered a job as a salesman in the men's department of two large department stores. Store A offered Carl a basic salary of \$6.25 per hour for a 40 hour week, and a commission of 6% of all his sales. Store B offered Carl a basic salary of \$8.50 an hour, with a 4 1/2 % commission on all his sales. Because of his experience in sales, Carl has a pretty good idea of how much clothing he can sell, but Carl doesn't know any math. Write a brief report Carl could use to determine which job offer to accept.
19. Elissa works 8 hour shifts at the Waffle House. She makes \$34.80 plus tips. Assume that each day her customers tip on average the same percent.
- (a) If she earned \$82.68 on a day when her customers ordered \$342 of food and drink, and \$112.64 on a day when her customers ordered \$556 worth of food and drink, what is the rate at which she receives tips?
- (b) With the above tipping rate, how much more food and drink would her customers have to order for Elissa to earn an extra \$16.12?
20. A congressman thinks up the following tax plan. If you make less than \$25,000 a year, the government pays you 8% of the difference between \$25,000 and what you earn. If you earn \$25,000 or more, you pay in taxes 8% of the difference between what you earn and \$25,000. You can think of receiving a tax check *from* the government as a *negative* tax.
- (a) One family earns \$37,000 more than another. What is the difference in the taxes they owe? Explain.
- (b) The Jones family pays \$8960 in taxes. If the Smith family earns \$31,000 less than the Jones family, how much will the Smith family pay in taxes?
21. Sometimes companies give volume discounts to good customers. A department store was having a sale, with a 15% discount on the total value in excess of \$75 of all goods purchased, and an extra 10% discount on the total value in excess of \$200 of all goods purchased.
- (a) Juanita paid \$40 more than Sharon for her purchases? How much more were they worth before discounts? Explain your answer.
- (b) Bruce paid \$350 for all the clothes he purchased. How much were they worth before discounts?